Quantum Modeling: from Coarse Graining to a Tower of Scales

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We present a family of methods which can describe complex behaviour in quantum ensembles. We demonstrate the creation of nontrivial (meta) stable states (patterns), localized, chaotic, entangled or decoherent, from the basic localized modes in various collective models arising from the quantum hierarchy described by Wigner-like equations. The advantages of such an approach are as follows: i). the natural realization of localized states in any proper functional realization of (Hilbert) space of states, ii). the representation of hidden symmetry of a chosen realization of the functional model describes the (whole) spectrum of possible states via the so-called multiresolution decomposition. Effects we are interested in are as follows: 1. a hierarchy of internal/hidden scales (time, space, phase space); 2. non-perturbative multiscales: from slow to fast contributions, from the coarser to the finer level of resolution/decomposition; 3. the coexistence of the levels of hierarchy of multiscale dynamics with transitions between scales; 4. the realization of the key features of the complex quantum world such as the existence of chaotic and/or entangled states with possible destruction in "open/dissipative" regimes due to interactions with quantum/classical environment and transition to decoherent states. The numerical simulation demonstrates the formation of various (meta) stable patterns or orbits generated by internal hidden symmetry from generic high-localized fundamental modes. In addition, we can control the type of behaviour on the pure algebraic level by means of properly reduced algebraic systems (generalized dispersion relations).